

Introduction to quantum physics

Ferreira, A.C.

June 2021

1 Introduction

These notes have an introduction to Quantum Physics. By "notes" I mean that I'll not go further into any deduction or deep explanation.

2 Atomism

2.1 Dalton's Atomic Theory

Note: In this book, we are using $\langle a \rangle$ for mean value and $\langle a^2 \rangle$ for mean square value of a .

By conservation of mass, any molecule behaves like this example:

$$M_{HO} = N(aM_H + bM_O) \quad (1)$$

Where M_{HO} is a molecule constituted by Hydrogen and Oxygen with a and b atoms of each, respectively.

Using the hypotheses of kinetic theory and Newtons laws,

$$p = \frac{1}{3} \frac{N}{V} m \langle v^2 \rangle = \frac{2}{3} \frac{N}{V} \langle \epsilon \rangle \quad (2)$$

where p and $\langle \epsilon \rangle$ are pressure and mean kinetic energy, respectively.

Remember that

$$N \langle \epsilon \rangle = U \text{ (internal energy)} \quad (3)$$

so

$$pV = \frac{2}{3} U \quad (4)$$

Considering two gases in thermal equilibrium

$$N_1 m_1 \langle v_1^2 \rangle = N_2 m_2 \langle v_2^2 \rangle. \quad (5)$$

The variation of entropy is

$$dS = \frac{dU}{T} + \frac{pdV}{T} \quad (6)$$

and applying to a mole of gas

$$\frac{3}{2} \frac{U(T)}{T} = R \quad (7)$$

where R is an universal and important constant known as universal constant of ideal gases.

Using equation 7 we have

$$U = \frac{3}{2} nRT \quad (8)$$

so

$$pV = nRT \quad (9)$$

This is the equation of state of ideal gases.

Using equation 2 we have

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_b T \quad (10)$$

where k_b is R/N_A and is known as the Boltzmann's constant.

Maxwell's distribution is

$$f(v_x, v_y, v_z) = \frac{dN_{v_x, v_y, v_z}}{N} = \left(\frac{m}{2\pi k_b T} \right)^{\frac{3}{2}} \exp\left(-\frac{1}{2} \frac{mv^2}{k_b T}\right) \quad (11)$$

and so the corresponding distribution of the module of the velocity is

$$\rho(v) = \frac{dN_v}{N} = 4\pi v^2 f(v^2) = \sqrt{\frac{2}{\pi}} \left(\frac{m}{k_b T} \right)^{\frac{3}{2}} v^2 \exp\left(-\frac{1}{2} \frac{mv^2}{k_b T}\right). \quad (12)$$

For the specific heat of ideal gases we have this set of equations:

$$\begin{aligned} dQ &= c_V dT \\ dQ &= c_p dT \end{aligned} \quad (13)$$

and using the first law of thermodynamics ($dU = dQ - dW$),

$$\begin{aligned} c_V &= \frac{dU_{mol}}{dT} \\ c_p &= c_V + R \\ \gamma &= \frac{c_p}{c_V} = \frac{c_V + R}{c_V} \end{aligned} \quad (14)$$

If q is the degree of freedom

$$\begin{aligned} U_{mole} &= \frac{1}{2} qRT \\ c_V &= \frac{q}{2} R \\ c_p &= \frac{q+2}{2} R \\ \gamma &= \frac{q+2}{q} \end{aligned} \quad (15)$$

For a monoatomic gas the kinetic energy of translation is

$$K_{trans} = \frac{1}{2}MV_{CM}^2 = \frac{1}{2}M(\dot{X}_{CM}^2 + \dot{Y}_{CM}^2 + \dot{Z}_{CM}^2) \quad (16)$$

and has three degrees of freedom. Its rotational kinetic energy, rotating around two axes, is

$$K_{rot} = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 \quad (17)$$

with five degrees of freedom, because each axes has three and there's a connection between them, so we count 1 less degree. So, for example, a particle rotating in three axes has 7 degrees of freedom.