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# MONOTONICITY AND THE ROY MODEL*

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In this note we study the implications on a bivariate normal Roy model of two sets of monotonicity hypotheses proposed recently by Manski and Pepper (*Econometrica*, Vol. 64 (2000), pp. 997–1011). In that simple context, we show that these hypotheses imply strong restrictions on the correlations structure between the decision and the rewards.

## 1 Introduction

For the past 40 years, labour economists have been studying the relationship between schooling and earnings (see Card (1999) for a recent review). Universally, researchers observe that average individual earnings increase with years of schooling. However, the causality of this association as well as its magnitude is a subject of debate. On the one hand, human capital theory predicts a positive and direct association between schooling and earnings, while on the other hand distinct theoretical arguments (for example signalling, Spence, 1973) suggest that an unobserved factor, i.e. ability, is positively correlated with both education and earnings. The latter point has important consequences in terms of empirical methodology. Indeed, estimates obtained from least squares estimates are plagued by ability and endogeneity biases and by measurement error (Griliches, 1977).

Despite the diversity of estimation methods used to provide unbiased estimates (twin studies, instrumental variable, propensity score matching), Ashenfelter *et al*. (1999) report in their meta-analysis that results are not sensitive to the technique used. Moreover, these methods are contentious; see Neumark (1999) for criticisms on the twin literature, or Bound *et al*. (1995) and Angrist *et al*. (1996) on instrumental variables estimators and their interpretation. Identification of the schooling effect crucially relies on the assumptions researchers impose on the data.

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Recently, several authors have attempted to characterize the magnitude of the returns to education without imposing such stringent constraints. For example, Manski (1990) relies solely on the distributions of the treatment and outcome variables to estimate bounds rather than point estimates. However, without further restrictions these 'worst-case' bounds are not precise enough to provide much information on the 'true' returns to education (Ginther, 2000).

In order to tighten the bounds, exclusion and/or monotonicity restrictions can be imposed. Manski and Pepper (2000) show that by imposing the following two restrictions, i.e. monotonicity in the selection (MTS) and monotonicity in the response to the treatment (MTR), the initial 'worst-case' bounds can be tightened. MTR is equivalent to recognizing that a higher level of treatment cannot have a negative effect on any individual's outcome. MTS implies that individuals who choose a higher level of education would receive above average rewards if they were otherwise reassigned to lower education levels.

This note compares the restrictions imposed by MTS and a weaker version of MTR on a selection model *à la* Roy (1951), where individuals choose the treatment maximizing their expected outcome. We limit ourselves to the bivariate case and we find that in general the MTS–MTR assumptions impose strong restrictions on the structure of the correlation between treatment decisions and rewards. Far from being a 'low-cost' estimation method in terms of restrictions, the MTS–MTR bounds can be, on the contrary, based on stringent untestable constraints of the correlation structure of the model.

## 2 Monotone Instrumental Variables

Following Manski and Pepper (2000), consider education as a treatment, which is possibly endogenously determined. The returns to education can be defined as the differences between the population means for earnings $Y(t)$ associated with $t$ years of schooling and for earnings $Y(s)$ associated with $s$ years of schooling:

$$D(t, s) = E[Y(t)] - E[Y(s)] \tag{1}$$

with $s < t$. When earnings are measured in logarithms, this difference is equivalent to a rate of return and is therefore directly comparable to estimates obtained by conventional methods.

As in any analysis of the effect of treatment, the difficulty is to compare the different groups, as individuals are only observed in one state and no information on the outcomes in the counterfactual states is available. The analysis supposes then that the treatment chosen, measured in years of education, $Z$, is observed. Some additional information is available in a monotone instrumental variable $V$. $V$ is a monotone instrumental variable in the sense that for any two different values in its range, say $v_1$ and $v_2$, with $v_1 \leq v_2$,

$$E[Y(t)|V = v_1] \leq E[Y(t)|V = v_2] \tag{2}$$

Note that $Z$ can be a particularly convenient choice of such a monotone instrumental variable because it provides some hope that an estimate of the return can be obtained with a limited amount of extra information.

Manski and Pepper (2000) show that imposing some further structure on the latent distribution of rewards can lead to relatively tight intervals. Two sets of assumptions taken together proved to have some power in that respect. The first set of assumptions demands that the rewards depend in a monotonous fashion on the amount of education acquired (monotonicity in the response, MTR). On the other hand, the second set of assumptions requires that more able individuals would be rewarded better than less able individuals at any level of education (monotonicity in the selection, MTS).

MTS is based on the assumption that more able individuals earn more than less able individuals of the same educational level. As education attained is a function of unobserved ability, MTS is expressed in terms of education attained: an individual A with a higher level of education than an individual B is thought to be more able. Hence, at all levels of education, A would have had higher returns than B. Formally, the MTS assumption states that

$$u_2 \geq u_1 \Rightarrow E[Y(t)|Z = u_2] \geq E[Y(t)|Z = u_1] \tag{3}$$

for any possible value of $t$.

MTR assumes that for any given realization of the couples $[Y(t_2), Y(t_1)]$, $[y_\omega(t_2), y_\omega(t_1)]$ say, we have

$$t_2 \geq t_1 \Rightarrow y_\omega(t_2) \geq y_\omega(t_1) \tag{4}$$

That is, in our schooling context, more schooling has a non-negative effect on earnings.

At first glance, these two assumptions do not appear to be too restrictive and appear rather plausible. Furthermore imposing them allows us to obtain upper and lower bounds on $E[Y(t)]$, in terms of quantities easily measured with data.

For every treatment level, the expected wage for all individuals in the population can be bounded above and below. Hence the difference in earnings between two levels of educational attainment is simply equal to the difference between the upper bound for the higher level of schooling and the lower bound for the lower level of schooling.[1] In particular, it can then be shown that the difference $D(t, s)$ can be bounded above by a quantity that can be estimated (simply) from the observed data.

[1]Similarly, the lower bound can be defined as the difference between the lower bound for the higher level of schooling and the higher bound for the lower level of schooling.

## 3 Monotonicity and the Roy Model: the Bivariate Normal Case

The MTR and MTS assumptions are not without consequences. In particular, given the implied decision process that determines individual actions, these two assumptions limit the nature of the joint latent reward process in general. This is easily seen in the context of a simple Roy model with normal distribution of skills (see Heckman and Honoré, 1990).

### *3.1 Monotonicity and the Original Roy Model*

We consider first the simplest possible case where individuals have the choice between two options, i.e. two education levels 0 and 1. Furthermore we assume that the preferred education level is the one with the highest wage level. Such a prototypical model of choice between (in our case) education levels allows us to analyse easily the consequences of the MTS and MTR assumptions. In particular, in such a simple structure we show that imposing the two assumptions together reduces the range of behaviour that the Roy model can explain.

Without loss of generality we describe the latent wages for each education level as follows

$$\begin{aligned} Y(0) &= \beta_0 + \varepsilon_0 \\ Y(1) &= \beta_1 + \varepsilon_1 \end{aligned} \tag{5}$$

where $\beta_0$ and $\beta_1$ are two constants, and where $\varepsilon_0$ and $\varepsilon_1$ are random variables which describe the heterogeneity in the population. In what follows we assume a weaker version of MTR (WMTR), which requires only that

$$E[Y(0)] \leq E[Y(1)] \tag{6}$$

The decision between the two education options, represented by the binary indicator $Z$, is therefore such that

$$\begin{aligned} Z &= 0 \qquad \text{if } Y(0) > Y(1) \Leftrightarrow \varepsilon_0 - \varepsilon_1 > \beta_1 - \beta_0 \equiv \gamma_{10} \\ Z &= 1 \qquad \text{if } Y(1) > Y(0) \Leftrightarrow \varepsilon_0 - \varepsilon_1 < \gamma_{10} \end{aligned} \tag{7}$$

We assume further that the heterogeneous returns to education for each education level are jointly normally distributed such that

$$\begin{pmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_0 - \varepsilon_1 \end{pmatrix} \sim \mathrm{N}\left(0, \begin{pmatrix} \sigma_0^2 & \rho\sigma_0\sigma_1 & \sigma_0^2 - \rho\sigma_0\sigma_1 \\ . & \sigma_1^2 & \rho\sigma_0\sigma_1 - \sigma_1^2 \\ . & . & \sigma_0^2 + \sigma_1^2 - 2\rho\sigma_0\sigma_1 \end{pmatrix}\right) \tag{8}$$

The relevant conditional moments are easily calculated. For $k = 0$ or 1, we have

$$E[Y(k)|Z=0] = \beta_k + E(\varepsilon_k|\varepsilon_0 - \varepsilon_1 > \gamma_{10})$$
$$= \beta_k + \theta_k \frac{\varphi(\gamma_{10}/\Sigma)}{1-\Phi(\gamma_{10}/\Sigma)} \tag{9}$$
$$E[Y(k)|Z=1] = \beta_k + E(\varepsilon_k|\varepsilon_0 - \varepsilon_1 < \gamma_{10})$$
$$= \beta_k - \theta_k \frac{\varphi(\gamma_{10}/\Sigma)}{\Phi(\gamma_{10}/\Sigma)} \tag{10}$$

where $\gamma_{10} > 0$ under WMTR,

$$\theta_0 \equiv \frac{\sigma_0^2 - \rho\sigma_0\sigma_1}{(\sigma_0^2 + \sigma_1^2 - 2\rho\sigma_0\sigma_1)^{\frac{1}{2}}} \tag{11}$$
$$\theta_1 \equiv \frac{\rho\sigma_0\sigma_1 - \sigma_1^2}{(\sigma_0^2 + \sigma_1^2 - 2\rho\sigma_0\sigma_1)^{\frac{1}{2}}}$$

and

$$\Sigma \equiv (\sigma_0^2 + \sigma_1^2 - 2\rho\sigma_0\sigma_1)^{\frac{1}{2}} \tag{12}$$

Moreover MTS implies that

$$E[Y(0)|Z=0] < E[Y(0)|Z=1]$$
$$\Rightarrow \theta_0 \le 0 \Rightarrow \sigma_0^2 - \rho\sigma_0\sigma_1 \le 0 \Rightarrow \rho \ge \frac{\sigma_0}{\sigma_1} \tag{13}$$

and

$$E[Y(1)|Z=0] < E[Y(1)|Z=1]$$
$$\Rightarrow \theta_1 \le 0 \Rightarrow \rho\sigma_0\sigma_1 - \sigma_1^2 \le 0 \Rightarrow \rho \le \frac{\sigma_1}{\sigma_0} \tag{14}$$

Thus MTS imposes $0 < \sigma_0/\sigma_1 \le \rho \le \min(\sigma_1/\sigma_0, 1)$. Hence MTS implies that the latent distribution of rewards without education must be less variable than the latent distribution of rewards with some education, i.e. $\sigma_0 \le \sigma_1$. Furthermore, in order to satisfy MTS the correlation between rewards must be strictly positive. Finally, in the limit as $\sigma_0 \to \sigma_1$ imposing MTS and WMTR implies that all individuals acquire some education almost surely.[2]

[2]Formally, $\sigma_0 \to \sigma_1 \Rightarrow \rho \to 1 \Rightarrow \Sigma \to 0 \Rightarrow \gamma_{10}/\Sigma \to \infty \Rightarrow \Phi(\gamma_{10}/\Sigma) \to 1$ and $1-\Phi(\gamma_{10}/\Sigma) \to 0$.

### *3.2 Monotonicity and a Generalized Roy Model with Individual Heterogeneity*

The previous section's conclusion changes if the decision rule is extended to include some individual heterogeneity, say $\kappa + \eta$, where $\kappa$ is some constant and $\eta$ is a random variable. This can be understood as an individual specific fixed cost associated with the higher education level. Such an interpretation leads to the following decision rule:

$$\begin{aligned} Z &= 0 \qquad \text{if } Y(0) > Y(1) - \kappa - \eta \\ Z &= 1 \qquad \text{if } Y(0) < Y(1) - \kappa - \eta \end{aligned} \tag{15}$$

where $\eta \sim \mathrm{N}(0, \sigma_\eta)$. This is an instance of a generalized Roy model. The joint distribution of the unobservable becomes

$$\begin{pmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_0 + \eta - \varepsilon_1 \end{pmatrix} \sim \mathrm{N}\left(0, \begin{pmatrix} \sigma_0^2 & \rho\sigma_0\sigma_1 & \theta_0'\Sigma' \\ . & \sigma_1^2 & \theta_1'\Sigma' \\ . & . & \Sigma'^2 \end{pmatrix}\right) \tag{16}$$

where $\delta_0 = \mathrm{Corr}(\varepsilon_0, \eta)$ and $\delta_1 = \mathrm{Corr}(\varepsilon_1, \eta)$ and

$$\Sigma'^2 = \sigma_0^2 + \sigma_1^2 + \sigma_\eta^2 + 2\delta_0\sigma_0\sigma_\eta - 2\rho\sigma_0\sigma_1 - 2\delta_1\sigma_1\sigma_\eta \tag{17}$$

$$\theta_0' = \frac{\sigma_0^2 + \delta_0\sigma_0\sigma_\eta - \rho\sigma_0\sigma_1}{\Sigma'} \tag{18}$$

$$\theta_1' = \frac{\delta_1\sigma_1\sigma_\eta + \rho\sigma_0\sigma_1 - \sigma_1^2}{\Sigma'} \tag{19}$$

And MTS implies that

$$\begin{aligned} \theta_0' \le 0 &\Rightarrow \rho \ge \frac{\sigma_0}{\sigma_1} + \delta_0\frac{\sigma_\eta}{\sigma_1} \\ \theta_1' \le 0 &\Rightarrow \rho \le \frac{\sigma_1}{\sigma_0} - \delta_1\frac{\sigma_\eta}{\sigma_0} \end{aligned} \tag{20}$$

Thus the correlation between the latent rewards is now bounded above and below by quantities that can be of different signs:

$$\max\left\{\frac{\sigma_0}{\sigma_1} + \delta_0\frac{\sigma_\eta}{\sigma_1}, -1\right\} \le \rho \le \min\left\{\frac{\sigma_1}{\sigma_0} - \delta_1\frac{\sigma_\eta}{\sigma_0}, 1\right\} \tag{21}$$

In particular the correlation of importance here is $\delta_0 = \mathrm{Corr}(\varepsilon_0, \eta)$, i.e. the correlation between the cost of education and the wage without education. Indeed for values of $\sigma_1/\sigma_0$ close to one, a negative value for $\delta_1$ does not modify the usual upper bound on a correlation, while a positive value for $\delta_1$ does modify the upper bound and the existence of the correlation depends then on the value of $\delta_0$. For example, $\sigma_1/\sigma_0 = 1$, $\sigma_\eta/\sigma_1 = \sigma_\eta/\sigma_0 = 0.5$ and $\delta_0 = -0.5$

implies $\rho > 0.75$. For $\delta_1 > 0.5$, $\rho$ is not defined; for $\delta_1 = 0.5$, $\rho$ is exactly 0.75; for smaller positive values of $\delta_1$, $\rho$ is bounded below by 0.75 and above by a quantity less than 1, and for any negative $\delta_1$ we have $0.75 \le \rho \le 1$.

Furthermore, the restrictions, given by Vijverberg (1993), which ensure that the variance–covariance matrix of the unobservables is semi-definite positive, need to be verified, i.e.

$$\delta_0\delta_1 - [(1-\delta_0^2)(1-\delta_1^2)]^{\frac{1}{2}} \le \rho \le \delta_0\delta_1 + [(1-\delta_0^2)(1-\delta_1^2)]^{\frac{1}{2}} \tag{22}$$

For example, when $\delta_1 = -0.5$, semi-positive definitiveness requires that $\rho$ belongs to the interval $[-0.5, 1]$ which contains $[0.75, 1]$ where MTS is true, while if $\delta_1 = 0.5$, $\rho$ must belong to the interval $[-1, 0.5]$ and this does not include $\rho = 0.75$.

Alternatively, if we believe that the latent rewards are uncorrelated, i.e. $\rho = 0$, for the extended Roy model to satisfy MTS, it is necessary for the correlations between the fixed costs and the latent rewards to be such that

$$\delta_0 \le -\frac{\sigma_0}{\sigma_\eta} \text{ and } \delta_1 \le \min\left\{\frac{\sigma_1}{\sigma_\eta}, 1\right\} \tag{23}$$

Moreover, definiteness when $\rho = 0$ demands

$$\delta_0^2 + \delta_1^2 \le 1 \tag{24}$$

However, this is only satisfied when $\sigma_\eta$, the variance of the fixed costs, is large relative to the variance of the latent reward $Y(0)$, i.e. when $\sigma_\eta > \sigma_0$, and for fixed costs moderately correlated with $Y(0)$.

## 4 Concluding Remarks

This straightforward exercise clearly shows that the monotonicity assumptions that Manski and Pepper impose are not compatible in general with the decision process assumed by the original Roy model and its extensions. The Roy model assumes that individuals decide on the level of education that leads to the highest level of earnings. In the normal case, for a range of parameter values, this is at odds with the MTS hypothesis. The MTS hypothesis requires that better able individuals are on average better rewarded whatever their educational attainment, i.e. not only do they choose more education but even if education was not available to them they would obtain a higher reward on average than the average individual choosing the lower education level. The Roy model does not impose such requirement on the latent distribution of rewards; it only requires that individuals decide on the basis of the highest reward. Potentially, a better educated individual, if denied the chance of an education, may end up with a lower than average wage among the less educated group.

Hence the claims in Manski and Pepper (2000, footnote 8) that the 'MTS restriction is consistent with economic models of schooling choice' and that the 'MTR restriction is consistent with economic models of the production of human capital through schooling' may need to be made more precise in general.

Finally, our analysis implies that, in order to compare meaningfully the range obtained from the methodology proposed by Manski and Pepper and the point estimates obtained if we assume a generalized Roy model with jointly normal unobservables, we should verify that the estimates of the Roy model are consistent with the MTS and MTR assumptions. The conclusion to reach in the likely case where the estimated Roy model is not consistent with the MTS–MTR assumptions remains an open question.

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